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A Notational Problem in Particle and Rigid Body Kinematics

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Abstract

This paper describes a problem associated with the use of standard subscript (or superscript) notation in particle and rigid body kinematics. Although it usually occurs when subscript letters are separated by a slash, it also arises if the slash is omitted. The problem concerns the proper interpretation of the commonly-used symbols $\bar{v}_{P/C}$ and $\bar{a}_{P/C}$, where P and C denote points, and these two symbols represent, respectively, the first and second time-derivatives, in some base reference frame B, of the relative position vector $\bar{r}_{P/C}$ that locates P relative to C. The common interpretations of $\bar{v}_{P/C}$ and $\bar{a}_{P/C}$, as the velocity and acceleration of P with respect to or relative to C, are criticized as being misleading and substantially incorrect. The correct interpretations of these symbols are identified as simply the differences between the B velocities and B accelerations of P and C. Additional problems with interpretation arise when two or more reference frames are used in the analysis. In such instances, the importance of adding extra letters to indicate the reference frame dependence associated with these symbols is emphasized.

Introduction

Good notation is a tremendously valuable tool in the effective communication of scientific concepts, and hence in the engineering educational process. There is an art as well as a science in choosing appropriate symbols to adequately represent well-defined quantities. At the same time, there is also a tradeoff that must be made between simplicity and complexity in deciding what symbols are most appropriate. The best symbols are those that are as simple as possible without losing their ability to convey what is actually being represented without confusion.

One of the most commonly-used notational conventions in particle and rigid body kinematics is the use of subscript (or superscript) letters. These letters usually occur in ordered pairs, they correspond to points or reference frames, and they may or may not be separated by a slash. For example, in particle kinematics, the symbol $\overline{r}_{P/C}$ is often used to represent the relative position vector locating point P relative to some convenient reference point C. The symbols $\overline{v}_{P/C}$ and $\overline{a}_{P/C}$ are then often used to represent, respectively, the first and second time-derivatives of $\overline{r}_{P/C}$ relative to some base reference frame B. These two vectors are commonly interpreted as representing the velocity ($\overline{v}$) and acceleration ($\overline{a}$) of P with respect to or relative to C[1,2,3,4,5]. Similarly, in rigid body kinematics, the symbol $\overline{\omega}_{R/B}$ is often used to represent the angular velocity of the rigid body R relative to the reference frame (or rigid body) B[6,7]. The symbol $\overline{\alpha}_{R/B}$ is then introduced to represent the first time-derivative in B of $\overline{\omega}_{R/B}$, where $\overline{\alpha}_{R/B}$ is commonly interpreted as the angular acceleration of R relative to B[6,7].

The question naturally arises as to the appropriateness of this relatively standard notational convention, and the common interpretations of these symbols. Are these symbols sufficiently detailed to describe with precision

the quantities they are designed to represent, yet sufficiently simple to really furnish a shorthand or abbreviated way of expressing these quantities? And do these symbols lend themselves to a correct interpretation of the actual quantities they are designed to represent, or are they subject to common misinterpretations?

The overall objective of this paper is to re-examine briefly these five basic kinematic quantities, and to show that the subscript (slash) notation is a completely acceptable and excellent way to represent these quantities provided two qualifications are met. These qualifications are that (1) proper care must be exercised in the interpretation of these symbols, and (2) additional symbolic details must be added as needed when auxiliary reference frames are used in the analysis. In particular, the purpose here is to show that $\overline{r}_{P/C}$, $\overline{\omega}_{R/B}$, and $\overline{\alpha}_{R/B}$ are all quite satisfactory symbols for the quantities they represent, that their common interpretations are correct, and that they require no additional symbolic details even when auxiliary reference frames are used in the analysis. A further purpose is to show that although $\overline{v}_{P/C}$ and $\overline{a}_{P/C}$ are also quite satisfactory symbols for the quantities they actually represent, their common interpretations are often incorrect or grossly misleading, and they require additional symbolic details when auxiliary reference frames are used in the analysis.

Basic Definitions

Before focusing on the issue of whether the subscript slash notation is appropriate in any particular instance, it will be useful to recall briefly five basic definitions from particle and rigid body kinematics.

1. Relative position. The relative position vector at time t, $\overline{r}_{P/O}(t)$, is the vector that locates one point (particle) P relative to another point O (e.g., the origin of a base reference frame B:OXYZ; see Figure 1) at time

t^7. Note that this instantaneous, vector quantity is not reference frame dependent (i.e., it depends only on the simultaneous, instantaneous locations of the two points P and O).

2. Velocity. The velocity of P in B at time t, $\bar{v}_P(t)$, is the time-derivative in B of $\bar{r}_{P/O}$ at time t^7. Thus

$$\bar{v}_P(t) = \lim_{\Delta t \to 0} \left(\frac{\Delta \bar{r}_{P/O}}{\Delta t}\right) = \frac{d\bar{r}_{P/O}}{dt} \quad .$$

This is also an instantaneous, vector quantity. Note, however, that velocity is reference frame dependent because the displacement $\Delta\bar{r}_{P/O}$ which occurs during Δt occurs relative to B. Note further that in order for $\Delta\bar{r}_{P/O}$ to reflect this displacement in B, the origin O of the associated relative position vector $\bar{r}_{P/O}$ must be a point fixed in B.

3. Acceleration. The acceleration of P in B at time t, $\bar{a}_P(t)$, is the time-derivative in B of $\bar{v}_P$ at time t^7. Thus

$$\bar{a}_P(t) = \lim_{\Delta t \to 0} \left(\frac{\Delta \bar{v}_P}{\Delta t}\right) = \frac{d\bar{v}_P}{dt} \quad .$$

Again, this is an instantaneous, vector quantity that is reference frame dependent because $\Delta\bar{v}_P$, the associated change in the B velocity of P, occurs relative to B.

4. Angular velocity. The angular velocity of the rigid body R relative to the reference frame B at time t, $\bar{\omega}_R(t)$, is the limit, as Δt approaches zero, of the Euler equivalent rotation vector $\Delta\bar{\theta}_R$ that describes the change in orientation of R relative to B during the time-interval Δt, divided by Δt[4]. Thus

$$\bar{\omega}_R(t) = \lim_{\Delta t \to 0} \left(\frac{\Delta \bar{\theta}_R}{\Delta t}\right) \quad .$$

Hence, this is also an instantaneous, vector quantity that is reference frame dependent because the change in orientation $\Delta \bar{\theta}_R$ occurs relative to B.

5. Angular acceleration. The angular acceleration of R in B at time t, $\bar{\alpha}_R(t)$, is the time-derivative in B of $\bar{\omega}_R$ at time t[7]. Thus

$$\bar{\alpha}_R(t) = \lim_{\Delta t \to 0} \left(\frac{\Delta \bar{\omega}_R}{\Delta t}\right) = \frac{d\bar{\omega}_R}{dt} \quad .$$

Again, this is an instantaneous, vector quantity that is reference frame dependent because $\Delta \bar{\omega}_R$, the associated change in the B angular velocity of R, occurs relative to B.

Although four of these five kinematic quantities are reference frame dependent, it is common practice <u>not</u> to symbolically denote this fact if the reference frame relative to which these quantities are defined is obvious. This practice has been followed here, and exceptions to it are relatively rare in undergraduate engineering dynamics textbooks[7,8,9]. This simplification can lead to confusion, however, when auxiliary reference frames are used in the analysis, and such auxiliary reference frames are commonly encountered in typical dynamics courses for undergraduate engineering students.

When auxiliary reference frames are considered, a variety of notational conventions have been used to indicate the reference frame dependence of these kinematic quantities. The subscript letter "r" or the subscript abbreviation "rel" (to indicate the word "relative") is often added to the velocity or acceleration symbol[5,10,11,12], as are subscripts identifying the coordinate axes of the auxiliary reference frame with respect to which the kinematic quantity is defined[3,6,13,14]. Thus, for example, the velocity of point P

relative to the auxiliary reference frame A:Cxyz with origin C might be expressed as $(\bar{v}_P)_r$ or $(\bar{v}_P)_{rel}$ or $(\bar{v}_P)_{xyz}$. Although all three of these symbols are acceptable and easily interpreted, the last is clearly preferred when more than one auxiliary reference frame is used in the analysis because the particular reference frame relative to which the quantity is defined is specifically identified.

Another satisfactory and similar way to overcome this problem, and the one used by Kane[15] and others[7,8,9], is to add a single pre-subscript (or pre-superscript) letter to the velocity or acceleration symbol to indicate the reference frame relative to which the kinematic quantity is defined. Thus, for example, the velocity and acceleration of P in B, and the angular velocity and angular acceleration of R in B, could be expressed as ${}_B\bar{v}_P$, ${}_B\bar{a}_P$, ${}_B\bar{\omega}_R$, and ${}_B\bar{\alpha}_R$, respectively.

Another possible way to circumvent this problem would be to use the subscript (or superscript) slash notation[7]. Thus, for example, these same four kinematic quantitites could be expressed as $\bar{v}_{P/B}$, $\bar{a}_{P/B}$, $\bar{\omega}_{R/B}$, and $\bar{\alpha}_{R/B}$, respectively. This subscript slash notation is simple and non-controversial when applied to the angular velocity and angular acceleration vectors $\bar{\omega}_{R/B}$ and $\bar{\alpha}_{R/B}$. It is definitely not standard, however, when applied to the velocity and acceleration vectors $\bar{v}_{P/B}$ and $\bar{a}_{P/B}$. Instead, what is standard practice when using the subscript slash notation with both the velocity and acceleration symbols is (1) to replace the subscript letter B after the slash with a letter identifying a reference point (e.g., the origin O) that is fixed in the base reference frame B, and (2) to interpret these standard symbols (i.e., $\bar{v}_{P/O}$ and $\bar{a}_{P/O}$) as representing the velocity and acceleration of the point P relative to the reference point O rather than as the velocity and acceleration of P relative to the reference frame B. It is precisely this dif-

ference in the use of subscript slash notation that lies at the heart of the interpretational problem, and which motivated the writing of this paper.

Motion Relative to Auxiliary Reference Frames

Differences associated with the use and interpretation of subscript slash notation are most critical and easiest to understand when the motion of a point P relative to a base reference frame B:OXYZ with origin O is also conveniently expressed relative to an auxiliary reference frame A:Cxyz with origin C (see Figure 1). In this situation, the subscript slash notation is introduced immediately, naturally and without confusion when writing the relative position vector $\bar{r}_{P/C}$ that locates point P relative to point C. Using the parallelogram law of vector addition, this vector can be expressed as

$$\bar{r}_{P/C} = \bar{r}_{P/O} - \bar{r}_{C/O} \quad . \tag{1}$$

Taking the time-derivative in B of Eq. (1), and continuing to use the subscript slash notation augmented now by pre-subscript letters to avoid confusion, one obtains

$$\frac{d}{dt}_{B}(\bar{r}_{P/C}) = \frac{d}{dt}_{B}(\bar{r}_{P/O} - \bar{r}_{C/O}) = {}_{B}\bar{v}_{P} - {}_{B}\bar{v}_{C} = {}_{B}\bar{v}_{P/C} \quad . \tag{2}$$

Equation (2) indicates that the only logical interpretation of the symbol ${}_{B}\bar{v}_{P/C}$ derived in this manner is that it represents the difference in the B velocities of points P and C. If the auxiliary reference frame A is not introduced and the pre-subscript letter B is therefore not needed, this same interpretation of $\bar{v}_{P/C}$ is again proper and reasonable. In neither case, however, is it acceptable to interpret ${}_{B}\bar{v}_{P/C}$ as the velocity of P relative to C. This observation follows because the velocity of a point occurs relative

to a reference frame and not relative to a (reference) point.

Before leaving Eq. (2), note that if points P and C are fixed in the same rigid body A, and if A rotates relative to B with angular velocity ${}_B\bar{\omega}_A$ and angular acceleration ${}_B\bar{\alpha}_A$, then Eq. (2) becomes

$$ {}_B\bar{v}_{P/C} = {}_B\bar{v}_P - {}_B\bar{v}_C = ({}_B\bar{\omega}_A \times \bar{r}_{P/C}) \quad . \tag{3} $$

This expression follows directly from the velocity analog of Chasle's theorem for the most general displacement of a rigid body[10]. It may be used to express the time-derivative in B of a unit vector $\bar{u}_A$ fixed in A. This fundamental result[15] takes the form

$$ \frac{d}{{}_B dt}(\bar{u}_A) = ({}_B\bar{\omega}_A \times \bar{u}_A) \quad . \tag{4} $$

Taking the time-derivative in B of Eq. (2), and continuing to use the subscript slash notation with pre-subscript letters, one obtains

$$ \frac{d}{{}_B dt}({}_B\bar{v}_{P/C}) = \frac{d}{{}_B dt}({}_B\bar{v}_P - {}_B\bar{v}_C) = {}_B\bar{a}_P - {}_B\bar{a}_C = {}_B\bar{a}_{P/C} \quad . \tag{5} $$

Equation (5) indicates that the only logical interpretation of the symbol ${}_B\bar{a}_{P/C}$ derived in this manner is that it simply represents the difference in the B accelerations of points P and C. If the auxiliary reference frame A is not introduced and the pre-subscript letter B is therefore not necessary, this same interpretation of $\bar{a}_{P/C}$ is again both proper and reasonable. In neither case, however, is it acceptable to interpret ${}_B\bar{a}_{P/C}$ as the acceleration of P relative to C. This observation is true because the acceleration of a point occurs relative to a reference frame and not relative to a point.

The common misinterpretations of $\bar{v}_{P/C}$ and $\bar{a}_{P/C}$, as the velocity and acceleration of P relative to C, make no sense because velocity and acceleration are kinematic properties of points that are reference frame dependent rather than reference point dependent. The velocity and acceleration of point P do not depend in any way on the reference point O fixed in the base reference frame B relative to which the location of P is specified. Any point O' that is fixed in B would suffice for this purpose[7,8]. It is simply meaningless to even talk about the velocity or acceleration of one point relative to another point, and this problem is not circumvented by adding a qualifying clause that describes how an auxiliary reference frame A, moving with the reference point C, is changing its orientation relative to the base reference frame B. Such a clause simply fails to correct the fundamental misconception that a point can have velocity and acceleration relative to another point. The only proper interpretation of the symbols ${}_B\bar{v}_{P/C}$ and ${}_B\bar{a}_{P/C}$ is that they represent the differences in the B velocities and B accelerations of points P and C. Anything else is at best misleading and unfortunate, and at worst, improper and substantially incorrect.

If the reference frame dependence is not indicated and more than one reference frame is used in the analysis, the symbols $\bar{v}_{P/C}$ and $\bar{a}_{P/C}$ are incomplete and dangerously inadequate. This is true because the differences in the B velocities and B accelerations of points P and C are generally not equal, respectively, to the differences in the A velocities and A accelerations of these same two points (See subsequent eqs. (14) and (16).). Again, even if the reference frame dependence is included in the symbol (e.g., by adding the appropriate pre-subscript letter), the interpretations of ${}_B\bar{v}_{P/C}$ and ${}_B\bar{a}_{P/C}$ as representing the velocity and acceleration of P relative to C in B are misleading and incorrect for the same reasons as mentioned previously. There

simply is no such thing as the velocity or the acceleration of one point relative to another point in any reference frame, unless such descriptions are meant to imply the velocity and acceleration differences in that reference frame. If this implication is intended, then the use of the word "relative" in this sense is inconsistent with its use in the definitions of velocity and acceleration, and therefore misleading, confusing, and unfortunate, particularly from the student's perspective.

To understand why this interpretational problem exists, consider again the motion of P expressed relative to both the base reference frame B:OXYZ with origin O, and relative to the right-handed, orthogonal auxiliary reference frame A:Cxyz with origin C. Let $\bar{i}$, $\bar{j}$, and $\bar{k}$ represent three mutually perpendicular unit vectors directed in the positive x, y, and z directions of A, respectively.

Consider first the case when A translates relative to B, or when ${}_B\bar{\omega}_A = \bar{0}$. Projecting $\bar{r}_{P/C}$ onto A, Eq. (1) can be written in the form

$$\bar{r}_{P/C} = x\bar{i} + y\bar{j} + z\bar{k} = \bar{r}_{P/O} - \bar{r}_{C/O} \quad . \tag{6}$$

Differentiating Eq. (6) once with respect to time in B, and using Eq. (4) to show that

$$\frac{{}_B d\bar{i}}{dt} = \bar{0} = \frac{{}_B d\bar{j}}{dt} = \frac{{}_B d\bar{k}}{dt} \quad ,$$

one obtains the expression

$$\frac{{}_B d}{dt}(\bar{r}_{P/C}) = \dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k} = {}_B\bar{v}_P - {}_B\bar{v}_C = {}_B\bar{v}_{P/C} \quad . \tag{7}$$

Noting that

$$\dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k} = \frac{d}{{}_A dt}(\bar{r}_{P/C}) = {}_A\bar{v}_{P/C} = {}_A\bar{v}_P - {}_A\bar{v}_C \,, \tag{8}$$

and recalling that ${}_A\bar{v}_C = \bar{0}$, Eq. (7) can be re-expressed as

$${}_B\bar{v}_{P/C} = {}_A\bar{v}_P \,. \tag{9}$$

Hence, for the case when A translates relative to B, the difference in the B velocities of points P and C, or ${}_B\bar{v}_{P/C}$, is actually equal to ${}_A\bar{v}_P$, the velocity of P in A. If one now incorrectly interprets the symbol ${}_A\bar{v}_P$ as the velocity of P relative to the reference point C fixed in A, then a rationale for the common misinterpretation of the symbol ${}_B\bar{v}_{P/C}$ has been identified.

Differentiating Eq. (7) once with respect to time in B, and again using Eq. (4), one obtains

$$\frac{d}{{}_B dt}({}_B\bar{v}_{P/C}) = \ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k} = {}_B\bar{a}_P - {}_B\bar{a}_C = {}_B\bar{a}_{P/C} \,. \tag{10}$$

Noting that

$$\ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k} = \frac{d^2}{{}_A dt^2}(\bar{r}_{P/C}) = \frac{d}{{}_A dt}({}_A\bar{v}_{P/C}) = {}_A\bar{a}_P - {}_A\bar{a}_C, \tag{11}$$

and recalling that ${}_A\bar{a}_C = \bar{0}$, Eq. (10) can be re-expressed as

$${}_B\bar{a}_{P/C} = {}_A\bar{a}_P \,. \tag{12}$$

Hence, for the case when ${}_B\bar{\omega}_A = \bar{0}$, the difference in the B accelerations of P

and C, or ${}_B\bar{a}_{P/C}$, is actually equal to ${}_A\bar{a}_P$, the acceleration of P in A. If one then incorrectly interprets ${}_A\bar{a}_P$ as the acceleration of P relative to the reference point C fixed in A, a similar rationale for the common misinterpretation of ${}_B\bar{a}_{P/C}$ has been identified.

Consider next the case when A rotates relative to B with angular velocity ${}_B\bar{\omega}_A$ and angular acceleration ${}_B\bar{\alpha}_A$. Differentiating Eq. (6) once with respect to time in B and using Eq. (4), one obtains

$$\frac{d}{dt}({}_B\bar{r}_{P/C}) = (\dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k}) + [{}_B\bar{\omega}_A \times (x\bar{i} + y\bar{j} + z\bar{k})] = {}_B\bar{v}_{P/C}. \qquad (13)$$

Using Eqs. (6) and (8), Eq. (13) can be re-expressed as

$${}_B\bar{v}_{P/C} = {}_A\bar{v}_{P/C} + ({}_B\bar{\omega}_A \times \bar{r}_{P/C}) \quad . \qquad (14)$$

This fundamental relationship is the well-known two reference frame differentiation formula[15]. It expresses the important fact that ${}_B\bar{v}_{P/C}$, the B derivative of $\bar{r}_{P/C}$, or the difference in the B velocities of P and C, is not generally equal to ${}_A\bar{v}_{P/C}$, the A derivative of $\bar{r}_{P/C}$, or the difference in the A velocities of these same two points.

Recalling that ${}_A\bar{v}_C = \bar{0}$, Eq. (13) can be re-written as

$${}_B\bar{v}_{P/C} = {}_A\bar{v}_P + ({}_B\bar{\omega}_A \times \bar{r}_{P/C}) \quad . \qquad (14)$$

Hence, for the general case when A rotates relative to B, the symbol ${}_B\bar{v}_{P/C}$ is properly interpreted as the difference in the B velocities of P and C, and this difference is equal to the sum of two vectors, one of which is ${}_A\bar{v}_P$, the velocity of P in A.

Continuing the process and differentiating Eq. (13) once with respect to time in B, the resulting equation takes the form

$$\frac{d}{dt}\left({}_B\bar{v}_{P/C}\right) = \left(\ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k}\right) + \left[{}_B\bar{\omega}_A \times \left(\dot{x}\bar{i} + \dot{y}\bar{j} + \dot{z}\bar{k}\right)\right]$$
$$+ \left({}_B\bar{\alpha}_A \times \bar{r}_{P/C}\right) + \left[{}_B\bar{\omega}_A \times \frac{d}{dt}\left(\bar{r}_{P/C}\right)\right] = {}_B\bar{a}_{P/C} \quad . \qquad (15)$$

Using Eqs. (8), (11), and (14), and again recalling that ${}_A\bar{v}_C = \bar{0} = {}_A\bar{a}_C$, Eq. (15) can be re-expressed as

$${}_B\bar{a}_{P/C} = {}_A\bar{a}_P + 2\left({}_B\bar{\omega}_A \times {}_A\bar{v}_P\right) + \left({}_B\bar{\alpha}_A \times \bar{r}_{P/C}\right) + \left[{}_B\bar{\omega}_A \times \left({}_B\bar{\omega}_A \times \bar{r}_{P/C}\right)\right] . \qquad (16)$$

Hence, in the general case when A rotates relative to B, the symbol ${}_B\bar{a}_{P/C}$ is properly interpreted as representing the difference in the B accelerations of P and C, and this difference is equal to the sum of four vectors, where only one of these vectors $\left({}_A\bar{a}_P\right)$ is the acceleration of P in A. Clearly, for this general case, it is grossly misleading to interpret ${}_B\bar{a}_{P/C}$ as the acceleration of P relative to C in B or in any other reference frame. Similarly, it is equally as misleading to imply that the Coriolis acceleration, $2\left({}_B\bar{\omega}_A \times {}_A\bar{v}_P\right)$, associated with the motion of P in B is a unique quantity when, in fact, it varies with the motion of the auxiliary reference frame A relative to which it is defined.

Summary

The subscript slash notation is a useful shorthand way to express various quantities in particle and rigid body kinematics. However, such notation must be carefully interpreted, and quantities that are reference frame dependent must be represented by symbols that indicate this dependence whenever more than one reference frame is used in the analysis and confusion could otherwise

result.

Time-derivatives in any reference frame of relative position vectors such as $\overline{r}_{P/C}$ are naturally expressed using the symbols $\overline{v}_{P/C}$ and $\overline{a}_{P/C}$ which incorporate the same subscript slash notation. However, these velocity and acceleration quantities should definitely not be interpreted as they sometimes are in undergraduate engineering dynamics courses and textbooks. These interpretations, as the velocity and acceleration of point P relative to another point C, are misleading and substantially incorrect. The inclusion of a qualifying clause stating something about the rotational motion, relative to a base reference frame B, of an auxiliary reference frame A that moves with the reference point C, simply fails to correct this fundamental misconception. Velocity and acceleration are kinematic properties of points that are reference frame dependent. They are in no way dependent upon the particular reference point O fixed in the base reference frame B relative to which the location of point P is specified.

These facts need to be emphasized, particularly to students who are studying this material for the first time. Although the authors of all the undergraduate engineering dynamics textbooks cited here seem to be aware of the reference frame dependence of velocity, acceleration, angular velocity and angular acceleration, they have not all been equally judicious in their choice of symbols to represent these quantities, nor in their interpretive statements when discussing these symbols. Particle and rigid body kinematics furnish enough challenges for students without unnecessarily complicating matters by either using inappropriate or incomplete symbols, or by introducing misleading or incorrect interpretations of otherwise reasonable and appropriate symbols.

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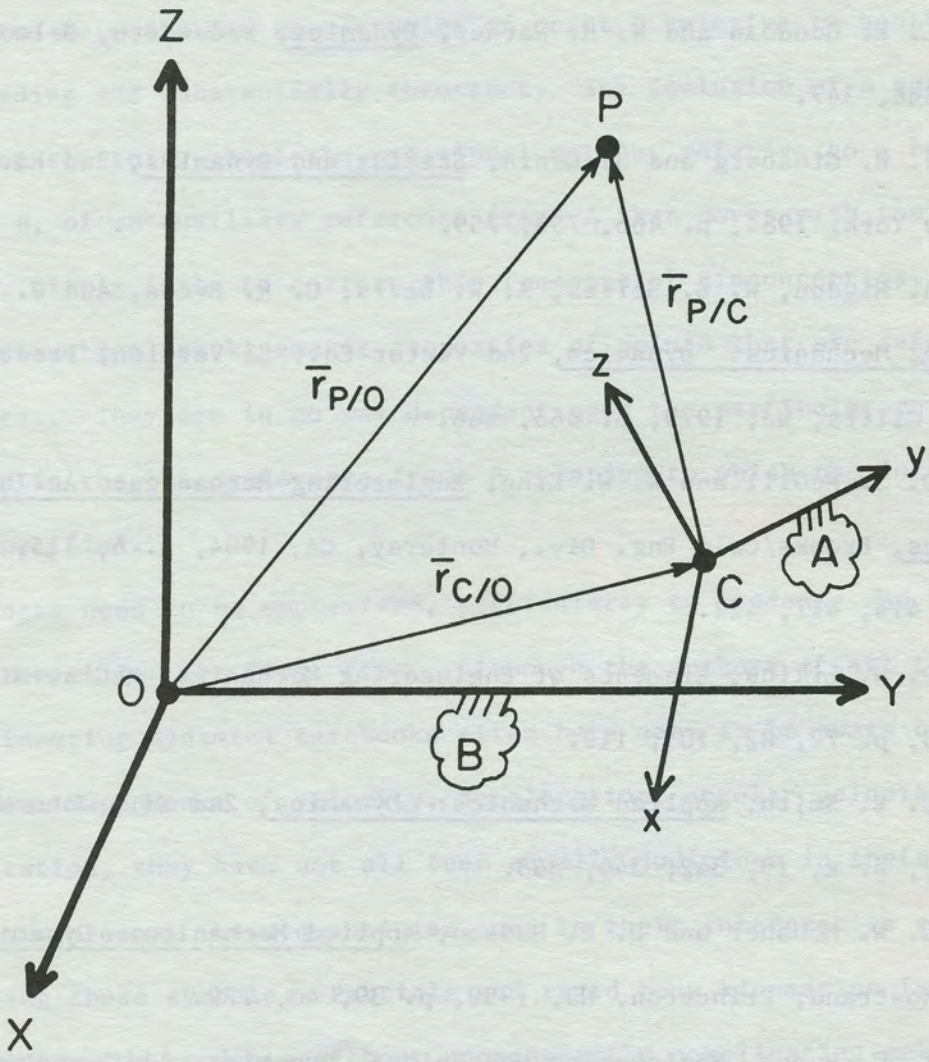

Figure 1. Point P, located relative to the origin O of the base reference frame B:OXYZ by $\bar{r}_{P/O}$, and located relative to the origin C of the auxiliary reference frame A:Cxyz by $\bar{r}_{P/C}$, where point C is located relative to O by $\bar{r}_{C/O}$.

Effect of Member Rotation on Nonlinear Behavior of a Simple Truss Made of a Highly Deformable Material

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ABSTRACT

An exact nonlinear analysis of the behavior of a simple symmetric two-member truss consisting of perfectly elastic material is presented. It is shown that the dimensionless deflection (deflection/original length of a member) is a function of only three dimensionless parameters: initial member orientation, dimensionless loading, and change in member orientation. Further, the last named quantity is a function of the first two. Numerical results from a parametric study are presented and compared with those obtained by elementary analysis.

INTRODUCTION

Many structural engineers are familiar with the approximation made in linearizing the exact curvature expression in analysis of beams, i.e.,

$$\kappa \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} \approx \frac{d^2y}{dx^2}$$

However, it is not widely recognized as to how important is the effect of member rotation in truss structures. The standard advanced mechanics of materials and structures texts with which the present investigator is familiar merely state that member rotation is

neglected, without giving any indication of the magnitude of the error ensuing from this assumption.

The purpose of this investigation is to provide an exact analysis of the problem of a simple, symmetric two-member truss and to compare the predictions of this analysis with the elementary one in which member rotation is neglected.

HYPOTHESES

To clarify and illustrate the importance of member rotation, the following simplifying engineering assumptions are made:

1. The initial geometry of the truss is defined as shown by solid lines in Fig. 1.
2. The upper support points remain fixed in location.
3. The joints of the truss are perfect, i.e. they do not deform and they are frictionless.
4. The loading is vertical and static.
5. The truss members are identical in length and cross-sectional area and are made of the same material.
6. The truss member material is linearly elastic.

Of course, one could relax these assumptions by incorporating complicating effects such as unsymmetric geometry, deformable supports, deformable joints with friction, and material nonlinearity. However, introduction of these complicating effects would cloud the major issue being addressed: that a simple truss structure can exhibit nonlinear force-displacement behavior strictly due to geometric rotation.

EXACT ANALYSIS

The geometry of the truss before and during loading is depicted schematically in Fig. 1.

From the geometry of Fig. 1, it can be seen that the exact expression for the vertical deflection δ_V is

$$\delta_V = (L + \delta) \cos(\alpha - \phi) - L \cos \alpha \tag{1}$$

Also

$$(L + \delta) \sin(\alpha - \phi) = L \sin \alpha \tag{2}$$

From elementary solid mechanics (strength of materials), the stretching displacement δ of a two-force member of initial length L is

$$\delta = \frac{FL}{AE} \tag{3}$$

where A ≡ cross-sectional area of member, E ≡ Young's modulus, and F ≡ axial force on member.

From the statics associated with Fig. 1, it is apparent that member force F is related to the applied load P as follows:

$$F = \frac{P}{2 \cos(\alpha - \phi)} \tag{4}$$

Combining Eqs. (2)-(4), one obtains

$$\left[1 + \frac{\Delta}{\cos(\alpha - \phi)}\right]^2 [1 - \cos^2(\alpha - \phi)] = \sin^2\alpha \tag{5}$$

where

$$\Delta \equiv \frac{P}{2AE} \tag{6}$$

Equation (5) is quartic in $\cos(\alpha - \phi)$ and is best solved for ϕ as a function of α and Δ by simple trial and improve procedure. Once ϕ is

known, one may use the following expression, obtained by combining Eqs. (1), (3), (4), and (6), to calculate δ_V:

$$\frac{\delta_V}{L} = \Delta + \cos(\alpha - \phi) - \cos \alpha \tag{7}$$

APPROXIMATE ANALYSIS TO OBTAIN THE ROTATION ANGLE

To avoid the solution of an unwieldy quartic equation in $\cos(\alpha - \phi)$ to obtain ϕ, an approximate solution for ϕ is desired. In the ensuing, such a solution is developed by using trigonometric expansions and then linearizing for small values of ϕ.

First, one obtains

$$\sin(\alpha - \phi) = \sin \alpha \cos \phi - \cos \alpha \sin \phi \simeq \sin \alpha - \phi \cos \alpha \tag{8}$$

Substituting the right side of Eq. (8) into Eq. (2) and solving for ϕ, one gets

$$\phi = \frac{1}{(L/\delta) + 1} \tan \alpha \tag{9}$$

Eqs. (3), (4), and (6) may be combined to obtain

$$L/\delta = (1/\Delta) \cos(\alpha - \phi) \tag{10}$$

However,

$$\cos(\alpha - \phi) = \cos \alpha \cos \phi + \sin \alpha \sin \phi \simeq \cos \alpha + \phi \sin \alpha \tag{11}$$

Thus,

$$L/\delta = (1/\Delta)(\cos \alpha + \phi \sin \alpha) \tag{12}$$

Substitution of Eq. (12) into Eq. (9) yields the following quadratic equation in ϕ:

$$\phi^2 + \left(\frac{\Delta}{\sin \alpha} + \cot \alpha\right)\phi - \frac{\Delta}{\cos \alpha} = 0 \tag{13}$$

The only valid solution of Eq. (13) is the one which yields a positive value of ϕ, namely,

$$\phi = -\frac{1}{2}\left(\frac{\Delta}{\sin\alpha} + \cot\alpha\right) + \left[\frac{1}{4}\left(\frac{\Delta}{\sin\alpha} + \cot\alpha\right)^2 + \frac{\Delta}{\cos\alpha}\right]^{\frac{1}{2}} \tag{14}$$

APPROXIMATE SOLUTIONS FOR VERTICAL DISPLACEMENT

Knowing ϕ from Eq. (14), one could use the exact expression, Eq. (7), for δ_V/L.

Making use of the approximation in Eq. (11), one could simplify Eq. (7) to the following:

$$\delta_V/L = \Delta + \phi \sin\alpha \tag{15}$$

Still another approximation for δ_V/L can be obtained by assuming that $L\cos\phi \simeq L$. Then, as shown in Fig. 2,

$$\delta_V = \delta/\cos(\alpha - \phi) \tag{16}$$

or, in view of Eq. (10),

$$\delta_V/L = \Delta/\cos^2(\alpha - \phi) \tag{17}$$

For purposes of comparison, the elementary solution obtained either from equilibrium or energy considerations with the member rotation (ϕ) neglected is

$$\frac{\delta_V}{L} = \frac{\Delta}{\cos^2\alpha} \tag{18}$$

Typical values of Δ achievable in the elastic range with various materials may be calculated from

$$\Delta_{EL} = \frac{\sigma_{EL}}{E}\cos(\alpha - \phi) \simeq \frac{\sigma_{EL}}{E}\cos\alpha \tag{19}$$

in view of Eqs. (4) and (6). Here, σ_{EL} is the elastic limit stress. Thus, for moderate values of cos α, Δ_{EL} may range from 0.001 for steel to 0.25 for soft rubber.

NUMERICAL RESULTS

For the way in which the rotation angle ϕ is defined (see Fig. 1), only positive real values are reasonable from a physical viewpoint. Fortunately, for practical combinations of initial inclination α and dimensionless load Δ, the exact quartic expression has only one positive root. Thus, there is no problem regarding uniqueness of solution. Numerical results for the exact solution of Eq. (5) and the approximate solution embodied in Eq. (14) are summarized in Table 1. From this table, it can be concluded that member rotation increases with both initial inclination α and dimensionless load Δ. Also, the approximation of Eq. (14) is quite accurate for very small dimensionless loads typical of metallic materials. However, as both α and Δ are increased, the overestimation of Eq. (14) relative to the exact value increases.

For the same values of α and Δ as used in Table 1, the dimensionless vertical deflection values obtained by using either the exact or approximate values of ϕ in Eqs. (7), (15), and (17) are summarized in Table 2. Of course, the exact result is given by Eq. (7) with the exact value of ϕ. Inspection of this table shows that when the approximate value of ϕ is used, Eq. (7) provides a very close upper bound to the solution and Eq. (17) gives a lower bound. These trends are also illustrated graphically in Fig. 3. Also, Fig. 3 shows that the vertical deflection versus load relation is nonlinear, representing a hardening type of spring behavior.

Fig. 3 also shows the elementary linear solution obtained either from equilibrium or energy considerations with the member rotation ϕ ignored, as given by Eq. (18). For values $\alpha = 45^\circ$ and $\Delta = 0.25$, the elementary solution overestimates the vertical deflection by 31.1%.

CONCLUSION

It is concluded that even for relatively small dimensionless loads, elementary truss theory, in which the effect of member rotation is neglected, can result in some error in predicting the deflection of an elastic truss. Member rotation induces a nonlinear stiffening effect, even though the member material has a linear stress-strain relation. Both exact (quartic equation) and approximate (quadratic equation) solutions for member rotation are provided. Also, it is shown that use of the approximate rotation in the exact deflection equation results in a close upper bound to the exact solution.

TABLE 1

Effect of Member Initial Angle and Dimensionless Load on Member Rotation

α, deg.	Δ	Member Rotation ϕ, deg. Exact, Eq. (5)	Approx., Eq. (14)	% Error
30	0.001	0.0382	0.0382	0
45	0.001	0.0808	0.0808	0
60	0.001	0.1969	0.1969	0
45	0.010	0.7831	0.7883	0.66
60	0.010	1.799	1.843	2.39
45	0.250	11.99	12.84	7.09

TABLE 2

Effect of Member Initial Angle and Dimensionless Load on Dimensionless Vertical Deflection δ_V/L

α, deg.	Δ	δ_V/L based on exact ϕ Eq. (7)	Eq. (15)	Eq. (17)	δ_V/L based on approx. ϕ Eq. (7)	Eq. (15)	Eq. (17)
30	0.001	0.001333	0.001334	0.001332	0.001333	0.001334	0.001332
45	0.001	0.001997	0.001997	0.001994	0.001997	0.001997	0.001994
60	0.001	0.003973	0.003976	0.003953	0.003973	0.003976	0.003953
45	0.010	0.01960	0.01966	0.01947	0.01966	0.01973	0.01946
60	0.010	0.03693	0.03719	0.03602	0.03759	0.03786	0.03593
45	0.250	0.3815	0.3980	0.3555	0.3895	0.4085	0.3488

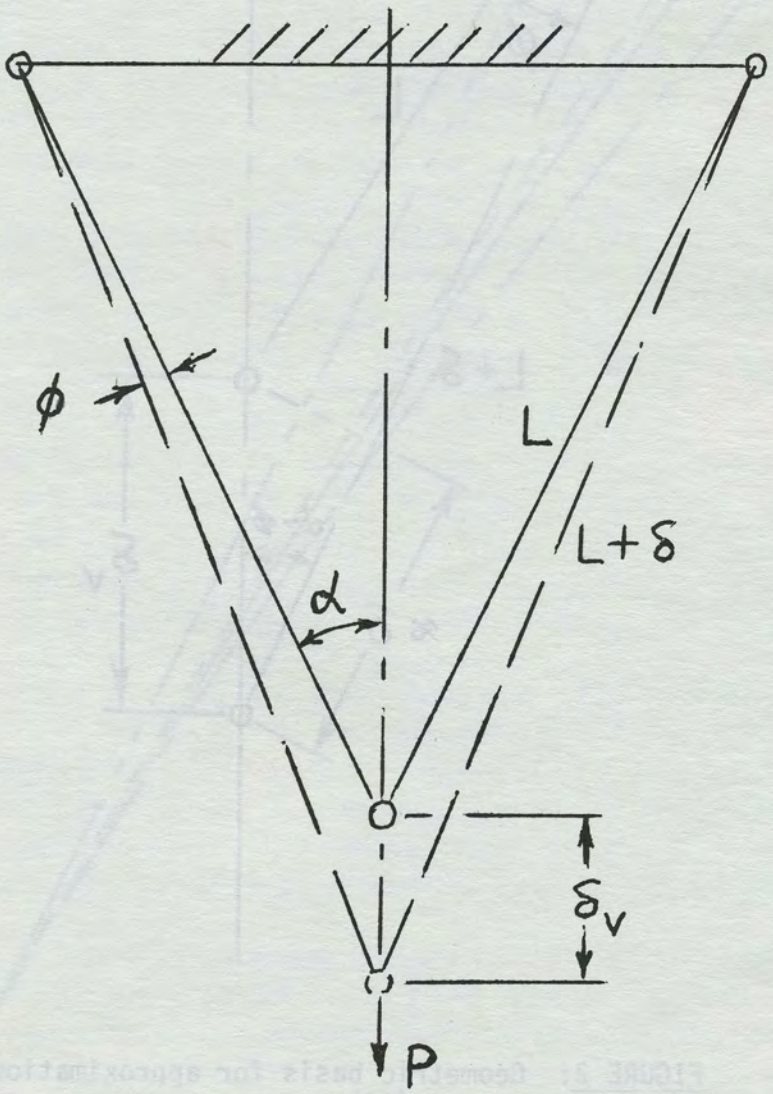

FIGURE 1: Truss geometry.

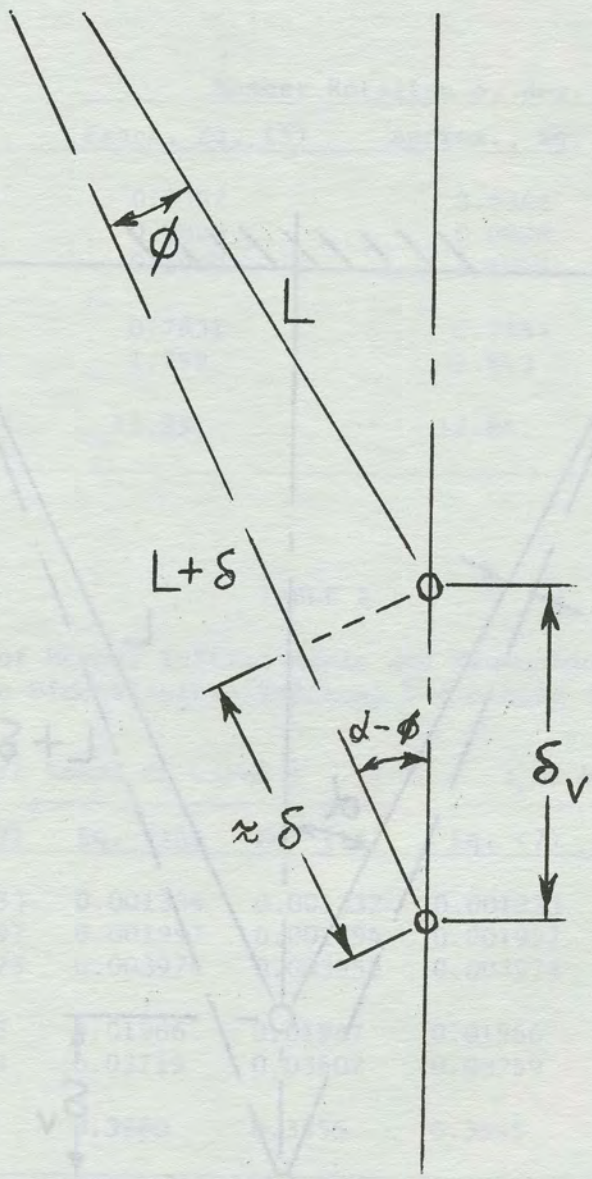

FIGURE 2: Geometric basis for approximation in Eq. (16).

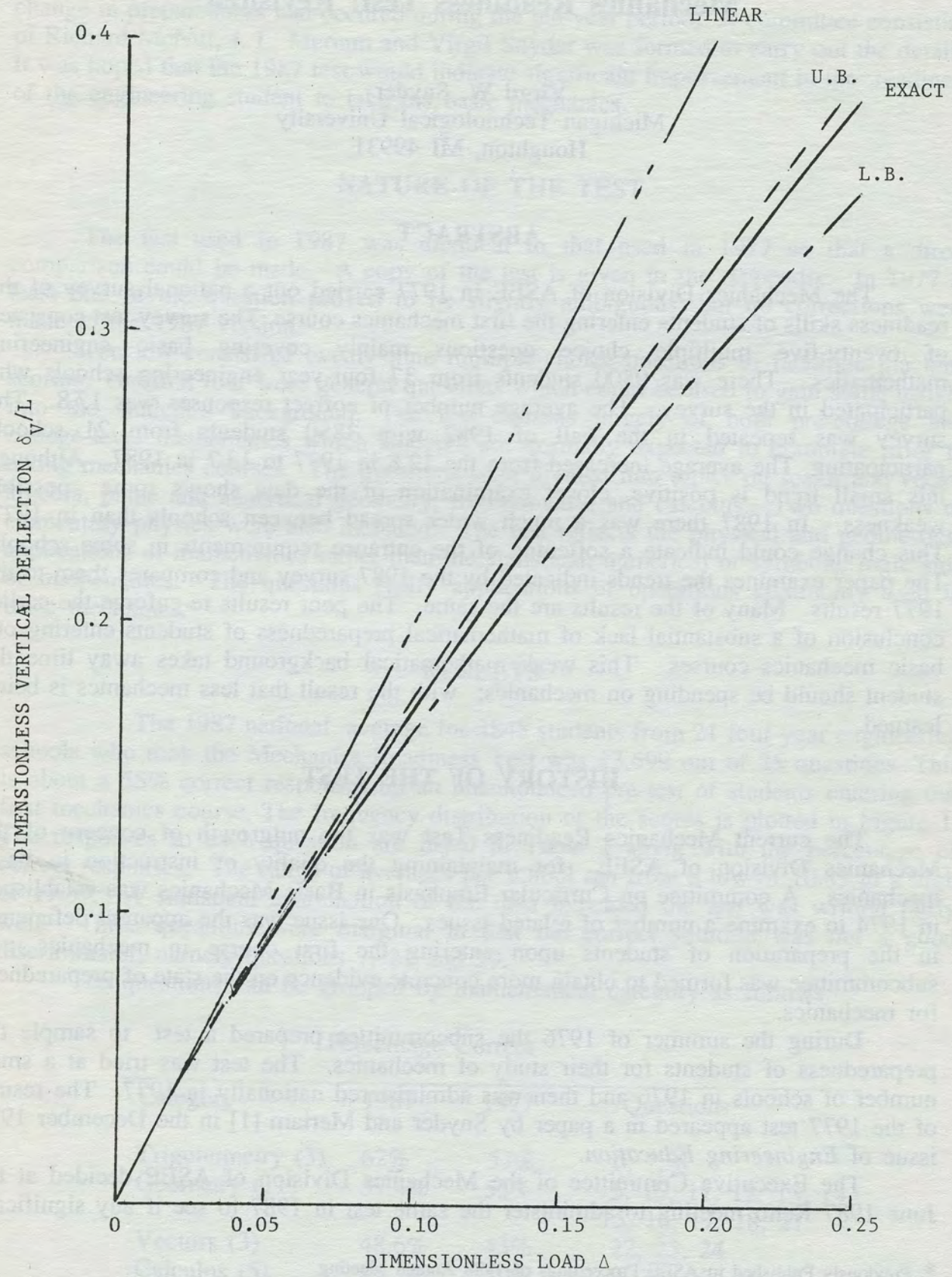

FIGURE 3: Dimensionless vertical deflection versus dimensionless load for α = 45 degrees.

Mechanics Readiness Test: Revisited*

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ABSTRACT

The Mechanics Division of ASEE in 1977 carried out a national survey of the readiness skills of students entering the first mechanics course. The survey test consisted of twenty-five muiltiple choice questions mainly covering basic engineering mathematics. There was 9500 students from 37 four-year engineering schools who participated in the survey. The average number of correct responses was 12.8. The survey was repeated in the Fall of 1987 with 3850 students from 21 schools participating. The average increased from the 12.8 in 1977 to 13.7 in 1987. Although this small trend is positive, closer examination of the data shows some possible weakness. In 1987 there was a much wider spread between schools than in 1977. This change could indicate a softening of the entrance requirements in some schools. The paper examines the trends indicated by the 1987 survey and compares them to the 1977 results. Many of the results are the same. The poor results re-enforce the earlier conclusion of a substantial lack of mathematical preparedness of students entering our basic mechanics courses. This weak mathematical background takes away time the student should be spending on mechanics; with the result that less mechanics is being learned.

HISTORY OF THE TEST

The current Mechanics Readiness Test was the outgrowth of concern of the Mechanics Division of ASEE for maintaining the quality of instruction in basic mechanics. A committee on Curricular Emphasis in Basic Mechanics was established in 1974 to examine a number of related issues. One issue was the apparent deficiency in the preparation of students upon entering the first course in mechanics. A subcommittee was formed to obtain more concrete evidence on the state of preparedness for mechanics.

During the summer of 1976 the subcommittee prepared a test to sample the preparedness of students for their study of mechanics. The test was tried at a small number of schools in 1976 and then was administered nationally in 1977. The results of the 1977 test appeared in a paper by Snyder and Meriam [1] in the December 1978 issue of *Engineering Education.*

The Executive Committee of the Mechanics Division of ASEE decided at the June 1987 Reno meeting to administer the same test in 1987 to see if any significant

* Previously Published in ASEE Proceedings of 1988 Annual Meeting

change in preparedness had occured during the ten-year period. A committee consisting of Richard McNitt, J. L. Meriam and Virgil Snyder was formed to carry out the details. It was hoped that the 1987 test would indicate significant improvement in the readiness of the engineering student to take his basic mechanics.

NATURE OF THE TEST

The test used in 1987 was identical to that used in 1977 so that a direct comparison could be made. A copy of the test is given in the Appendix. In 1977 at least one of the question proved to be slightly ambiguous, but no corrections were made to the 1987 version.

The test consist of twenty nine multiple-choice questions to facilitate machine scoring. The first four were general questions which could be used to gain some insight into the student's background. The test consists largely of both pre-college and college-level mathematics which students are normally expected to complete prior to taking mechanics courses. The questions were divided into topics on scalar and vector algebra, plane and analytical geometry, trigonometry, and calculus. Two questions of elementary physics were also included. The test reflects the physical and geometrical applications of mathematics rather than the statistical, numerical or symbolic logic side of mathematics. The questions typify applications or operations commonly used in basic mechanics.

RESULTS

The 1987 national average for 3848 students from 21 four-year engineering schools who took the Mechanics Readiness Test was 13.699 out of 25 questions. This is about a 55% correct responses on an unannounced pre-test of students entering our first mechanics course. The frequency distribution of the scores is plotted in Figure 1. The responses to each question are listed in Table 1 along with the percentage of correct responses. The range of averages per school ran from a low of 10.83 to a high of 17.39. A statistical examination of the data indicated the test was written fairly well. Three questions were marginal in that the correct solution was not a good discriminator, namely questions 7, 21 and 29.

The questions can be grouped by mathematical category as follows:

	Percentage Correct		
Category	1987	1977	Questions
Trigonometry (3)	62%	57%	6, 7, 8
Geometry (11)	57.4%	55%	5, 10, 11, 12, 13, 14 15, 16, 17, 18, 21
Vectors (3)	48.6%	43%	22, 23, 24
Calculus (5)	46.1%	40%	9, 25, 26, 27, 29

These subdivisions are by no means independent of one another as elements of algebra and geometry run through most of them. The results indicate about a 5 to 6 percent gain over the 1977 results except in geometry.

Question	Total Number	Responses 1	2	3	4	5	Answer	Percentage Correct 1987	1977
5	3272	31	35	3063	51	91	3	93.6	92.9
6	3267	2362	522	145	40	198	1	72.3	68.9
7	3177	389	1158	662	201	767	3	20.8	18.5
8	3274	171	8	59	3003	33	4	91.7	87.4
9	3249	304	161	291	1838	654	4	56.6	46.8
10	3061	326	1277	411	505	641	2	41.7	37.0
11	3068	264	1634	628	230	311	2	53.3	42.9
12	3251	2972	30	139	63	47	1	91.4	88.4
13	3159	71	1342	603	575	568	2	42.5	40.2
14	3214	177	1389	435	525	715	2	43.2	37.6
15	3226	244	223	184	2022	553	4	62.7	48.7
16	3272	60	19	3057	108	28	3	93.4	88.3
17	3240	532	46	166	79	2417	5	74.6	72.6
18	3093	1261	580	718	158	386	1	40.8	35.0
19	3152	174	2018	394	149	417	2	64.0	62.5
20	3257	1086	1956	76	67	72	2	60.1	56.5
21	3163	384	401	956	817	605	3	30.2	27.9
22	3224	2078	99	479	213	355	1	64.5	58.8
23	3142	795	1313	547	330	156	2	41.8	36.7
24	3100	1209	1177	290	200	224	1	39.0	33.2
25	3163	260	1480	307	506	610	2	46.8	35.0
26	3198	46	2411	154	280	306	2	75.4	66.1
27	3127	838	398	328	1156	407	4	37.0	31.0
28	3083	208	360	390	491	1634	5	53.0	49.9
29	3073	1009	1141	359	407	156	4	13.2	17.6

Table 1. Response Frequency to Each Question

DISCUSSION OF RESULTS

The results were quite similar to those in 1977. Some of the same remarks are repeated here for completeness. Considering the prerequisite mathematical background required for our first mechanics course, the 55% score seems very low. The authors of the test feel that an average score of at least 75% should be expected. The dismal results on this test substantiate the allegations that our students as a group

are seriously deficient in their understanding and ability to use even elementary tools of mathematics. The results have not improved significantly over the past decade. For a teacher of mechanics, it is hard to believe that 30% of the students could not determine the sine of an angle for a 3-4-5 triangle (Question 6). To cite a few other examples of the dismal results, almost 60% could not calculate the perpendicular distance from a point A to the y-axis in question 10; in question 13, 58% could not determine the cosine of the angle between the diagonal of a rectangular box and a side; and 70% could not approximate the area of a semi-circular ring in problem 21. Further space will not be used here to comment on the exceptionally poor results.

In all fairness to the students, one must not overlook the fact that students had no advanced notice of the test and most of them had not been exercising their grasp of mathematics during the summer. In 1977, students who took test in the spring did about two correct responses better than their counter part in the fall. In comparing the results of 1987 to 1977, not much new can be extracted. The 1987 students did preform slightly better increasing the average of correct responses from 12.8 to 13.7.

CONCLUSIONS

The state of preparedness of the entering students in mechanics is essentially no better in 1987 than in 1977. The recommendations make in the paper by Snyder and Meriam [1] clearly have not been effectively implemented. It is no wonder that students have difficulty learning mechanics in our basic courses, they have to spend much of their time re-learning elementary mathematics. The data for individual schools show some interesting trends. In 1977 the spread in average scores between the 37 schools was 11 to 15; in 1987 this spread was from 10.9 to 17.4 for the 21 schools. The reason for this great a change is unclear. The change could be attributed to the entrance requirements of entering freshman. The pressures to maintain enrollments may have softened the entrance requirements in some institutions. The results of the 1987 survey focus renewed concern on the problem of weakness in the understanding of elementary and basic mathematics for students entering engineering mechanics courses.

REFERENCES

[1] Snyder, V.W. and J.L. Meriam, "A Study of Mathematical Preparedness of Students: The Mechanics Readiness Test", *Engineering Education*, Dec. 1978, pp 261-267.

APPENDIX

MECHANICS READINESS TEST

Sponsored by the Mechanics Division of American Society for Engineering Education.

INSTRUCTIONS

A. Your score on this test will be used to help you and your instructor evaluate your readiness to study mechanics.

B. Notes, references, calculator, or slide rule may **not** be used.

C. Time limit for the test—45 minutes.

D. Print your name on the answer sheet. Fill in the appropriate space on the answer sheet provided for the *one correct alternative.* Make no marks on the test proper and return all exam pages and the IBM answer sheet to your instructor.

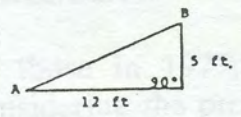
1. Your intended engineering major is:
   (1) Chemical Engineering
   (2) Civil Engineering
   (3) Electrical Engineering
   (4) Mechanical or Aeronautical Engineering
   (5) Other or not certain

2. Your last drawing or graphics course was in:
   (1) High School
   (2) College
   (3) None

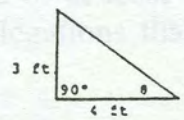
3. If your school is on a semester basis, how many calculus courses have you completed?
   (1) 0
   (2) 1
   (3) 2
   (4) 3
   (5) 4 or more

4. If your school is on a quarter basis, how many calculus courses have you completed:
   (1) 0
   (2) 1
   (3) 2
   (4) 3
   (5) 4 or more

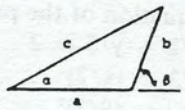
5. The length of AB is
   (1) 10 ft
   (2) $\sqrt{119}$ ft
   (3) 13 ft
   (4) 17 ft
   (5) None of the above



6. The value of sin θ is
   (1) 0.6
   (2) 0.75
   (3) 0.8
   (4) 3
   (5) None of the above



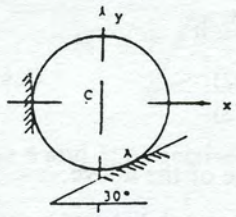
7. The length of side c of the triangle shown is:
   (1) $\sqrt{a^2 + b^2 + 2ab \sin \alpha}$
   (2) $\sqrt{a^2 + b^2 - 2ab \cos \beta}$
   (3) $\sqrt{a^2 + b^2 + 2ab \cos \beta}$
   (4) $\sqrt{a^2 - b^2 - 2ab \cos \alpha}$



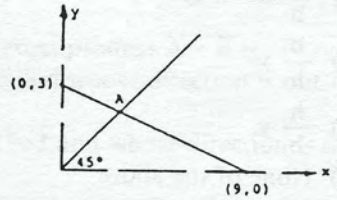
8. For an arbitrary angle θ
   (1) $\sin \theta + \cos \theta = 1$
   (2) $\sin \theta - \cos \theta = 1$
   (3) $\sin^2 \theta = 1 + \cos^2 \theta$
   (4) $\sin^2 \theta = 1 - \cos^2 \theta$
   (5) None of the above is true

9. As a close approximation, when an angle θ (expressed in radians) is very small, we may
   (1) replace $\cos \theta$ by zero
   (2) replace $\sin \theta$ by unity
   (3) replace $\cos \theta$ by θ
   (4) replace $\sin \theta$ by θ
   (5) None of the above

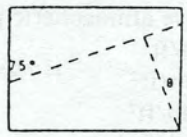
10. The 20-in diameter wheel with center at C rests against the two surfaces as shown. The perpendicular distance from point A to the y-axis is
   (1) 2.5 in
   (2) 5 in
   (3) 5.77 in
   (4) 8.66 in
   (5) None of the above



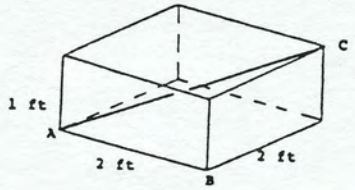
11. The x-coordinate of the point of intersection A of the two lines (which are not drawn to scale) is
   (1) 2
   (2) 2.25
   (3) 3
   (4) 3.5
   (5) None of the above



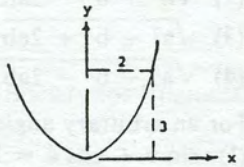
12. The two dotted lines in the rectangle are perpendicular to each other. Angle θ equals
   (1) 15°
   (2) 20°
   (3) 25°
   (4) 30°
   (5) None of the above



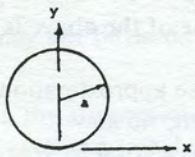
13. The rectangular box has a 2-ft by 2 ft-base and a height of 1 ft. The cosine of the angle between the diagonal AC and the edge AB is
   (1) 2/5
   (2) 2/3
   (3) $1/\sqrt{2}$
   (4) $2/\sqrt{5}$
   (5) None of the above



14. The equation of the parabola shown is
    (1) $x/2 + y/3 = 2$
    (2) $y/3 = (x/2)^2$
    (3) $x/2 = (y/3)^2$
    (4) $y = 2x^2 - 5$
    (5) None of the above

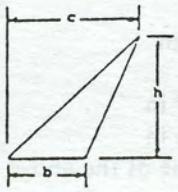


15. The equation of the circle shown is
    (1) $y^2 + (x - a)^2 = a^2$
    (2) $x^2 = y^2 + a^2$
    (3) $(x + y)^2 = a^2$
    (4) $x^2 + (x - a)^2 = a^2$
    (5) None of the above

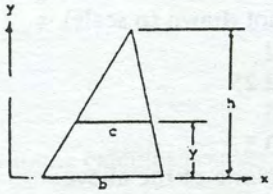


16. The area of a circle of radius r is
    (1) $(\pi r)^2$
    (2) $\pi^2 r$
    (3) πr^2
    (4) $2\pi r$
    (5) None of the above

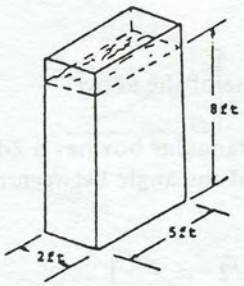
17. The area of the triangle is
    (1) $\frac{b+c}{2}\,h$
    (2) $b(b/2)$
    (3) $h(b/3)$
    (4) hb
    (5) None of the above



18. The horizontal distance c expressed in terms of y is
    (1) $\frac{b}{h}(h - y)$
    (2) $\frac{h}{b}(h - y)$
    (3) $\frac{b}{h}\,y$
    (4) $\frac{h}{b}\,y$
    (5) None of the above

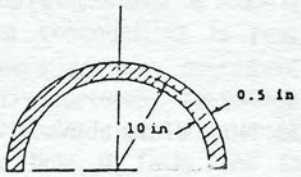


19. The tank shown is filled with salt water (specific weight 64 ib/ft³) to a depth of 8 ft. The pressure exerted by the water on the bottom of the tank above atmospheric pressure is
    (1) 64 lb/ft²
    (2) 512 lb/ft²
    (3) 640 lb/ft²
    (4) 14.7 lb/in²
    (5) None of the above

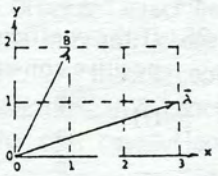


20. The mass of a man on earth is 72 kg. If the man traveled to the moon where the gravitational acceleration is 1/6 that on earth, his mass would be
    (1) 12 kg
    (2) 72 kg
    (3) 432 kg
    (4) 72 times the moon's gravitational acceleration
    (5) None of the above

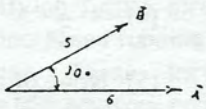
21. The shaded area of the half-circular ring is approximately
    (1) 3.14 in^2
    (2) 8 in^2
    (3) 16 in^2
    (4) 32 in^2
    (5) None of the above



22. The sum of vectors $\vec{A}$ and $\vec{B}$ is
    (1) a vector of magnitude 5
    (2) a vector of magnitude $\sqrt{10}$
    (3) a vector of magnitude $\sqrt{15}$
    (4) a vector whose components are (3,2)
    (5) None of the above



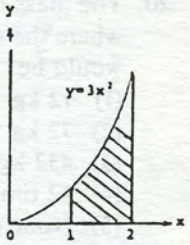
23. The magnitude of vectors $\vec{A}$ and $\vec{B}$ are 6 and 5, respectively. The dot product $\vec{A} \cdot \vec{B}$ is
    (1) a scalar whose magnitude is 15
    (2) a scalar whose magnitude is $15\sqrt{3}$
    (3) a scalar whose magnitude is 30
    (4) a vector whose magnitude is $15\sqrt{3}$
    (5) None of the above



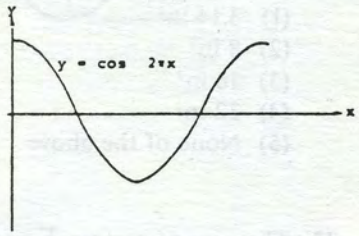
24. For the vectors in question 23, the cross product $\vec{A} \times \vec{B}$ is
    (1) a vector whose magnitude is 15 and whose direction is out from the paper
    (2) a vector whose magnitude of $15\sqrt{3}$ and whose direction is out from the paper
    (3) a vector whose magnitude is 15 and whose direction is into the paper
    (4) a scalar whose magnitude is 15
    (5) None of the above

25. If y = ln(sin x) where ln = $\log_e$, then dy/dx is
    (1) ln(cos x)
    (2) ctn x
    (3) tan x
    (4) sin x ln(cos x)
    (5) None of the above

26. The shaded area under the curve $y = 3x^2$ between $x = 1$ and $x = 2$ is
    (1) 1
    (2) 7
    (3) 8
    (4) 9
    (5) None of the above

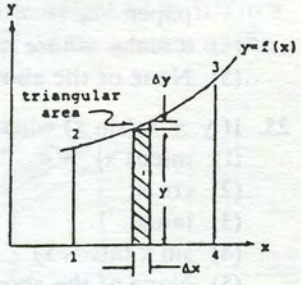


27. The slope of the curve $y = \cos 2\pi x$ at $x = \frac{1}{4}$ is
    (1) 0
    (2) 1
    (3) 2π
    (4) -2π
    (5) None of the above



28. If the coefficients a, b, and c of the equation $ax^2 + bx + c = 0$ are positive constants, the smaller of the two roots is
    (1) $-\frac{b}{a} + \frac{\sqrt{4ac - b^2 2}}{2a}$
    (2) $-\frac{b}{2} - \frac{\sqrt{b^2 - 4ac}}{2}$
    (3) $-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2b}$
    (4) $-\frac{1}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$
    (5) None of the above

29. To find the area 1-2-3-4 by integration, a vertical strip of area ΔA is chosen. In using only the cross-hatched area $y\,\Delta x$ for ΔA, the omission of the triangular area upon integration.
    (1) introduces only a small but finite error when the curvature is small
    (2) introduces a small but finite error regardless of the curvature
    (3) introduces a small error only if $y = f(x)$ is of second or higher degree in x
    (4) introduces no error
    (5) None of the above is true.



Mesomechanics: The Microstructure-Mechanics Connection

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INTRODUCTION

The structural mechanics discipline is primarily an outgrowth of applied mathematics. Historically, the structural or solid mechanician has described the mechanical behavior of materials as homogeneous and isotropic. Increasingly, the approaches have been modified to account for some anisotropy and, in more recent years, for some inhomogeneity.

The materials discipline is historically an outgrowth of chemistry, involving processing to achieve desired material properties. With the invention of the microscope, materials scientists began to study the interior structure of materials at the microscopic level and strived to establish the microstructure-property relationships. In the case of metals, necessary processing procedures to control the formation of the appropriate microstructure have been developed.

With the advent of engineered multiphase materials in recent years, most notably fiber composites, the prospect of controlling material microstructures and the resulting properties has been greatly enhanced. With composites, microstructure can be precisely controlled by the designer; and the resulting stiffness, strength, and durability properties can be predicted based on principles of mechanics. In addition to fiber composites, there are many other combinations of phase geometries and material types which can be designed to provide the desired properties. As performance requirements change, whether due to unique structural loading conditions or as a result of severe environments, the new trend is to consider the virtually unlimited possibilities of engineering a multitude of new materials with controlled microstructures and the predicted properties.

This new trend has caused both the structural mechanics community and the materials community to reexamine their individual roles and their interrelationship. A much closer collaboration of the two communities is now necessary in order to generalize and describe the various classes of microstuctures, to understand in detail how a particular class of microstructure responds to loads and how it fails, and to develop the processing necessary for the material to possess the desired microstructure. Drucker[1] describes this capability as " . . . design of the microstructure to give any desired combination of . . . macroscopic properties. . . ". According to Salkind[2], this concept of the design of materials is the real revolutionary impact of fiber composites.

During the past few years, there have been several related activities in the solid mechanics community aimed at identifying future directions for research, and advocating increased funding from the Government funding agencies. These include the meeting[3] organized by the NSF advisory committee held at SRI in November 1984; the ASME Applied Mechanics Division document on "Solid Mechanics Research Trends and Opportunities"[4], published in July 1985; the "Future Directions in Solid Mechanics Research" session held at the ASME Winter Annual Meeting in November 1985 in Miami; and related activities by the U. S. National Committee on Theoretical and Applied Mechanics.

These activities have identified a wide range of solid mechanics problem areas for future research, involving materials in bioengineering, electronic, geological and structural applications. Common to all these problem areas, there is a need to describe the materials' microstructure and to determine its influence on the materials' macro-response. The ability to establish the microstructure-property relationships based on principles of mechanics is clearly a major objective of such research.

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We at AFOSR believe that the recent activities of the solid mechanics community have provided substance for advocating increased research support, and are planning a funding enhancement for FY 1989 entitled: Mesomechanics:The Microstructure-Mechanics Connection. A planning team organized in the summer of 1986 consists of Dr. Anthony Amos, Maj. George Haritos and Maj. Joseph Hager of AFOSR; Dr. Nicholas Pagano of AFWAL Materials Laboratory; Dr. George Sendeckyj of AFWAL Flight Dynamics Laboratory; and Dr. Albert Wang of Drexel University, Visiting Scientist at AFOSR.

A special session was held at the ASME Winter Annual Meeting in December 1986 in Anaheim to discuss the planned effort and to solicit input from members of the solid mechanics research community. A presentation of the initial planning was made by Maj. Hager and Maj. Haritos, followed by discussions by invited panelists and attendees. Members of the panel included Dr. George Abrahamson, NSF Mechanics Advisory Committee; Professor James Dally, U. S. National Committee on Theoretical and Applied Mechanics; Professor Michael Carroll, ASME Applied Mechanics Division; Dr. Win Aung, NSF; Dr. Iradj Tadjbakhsh, ARO; Dr. Alan Kushner, ONR; Dr. James Welty, DOE; and Dr. Michael Salkind, AFOSR.

Many constructive comments and suggestions have been received from members of the solid mechanics community since the Anaheim meeting. The purpose of this paper is to summarize these inputs and current research plans.

THE ROLE OF MECHANICS IN THE DESIGN OF MATERIALS.

Development and application of new materials is now occurring at an accelerated rate. In the field of aerospace alone, one has already witnessed the phenomenal growth of fiber-reinforced composite materials in the past two decades. For fighter aircraft, for instance, composite usage was increased from 2% in the F-15, to 10% in the F-18, to 30% in the AV-8B, and is projected to be 40-60% in the ATF (Advanced Tactical Fighter). Efficient use of the remarkable stiffness-to-weight, strength-to-weight and other superior structural properties of fiber composites will expand even more rapidly if the material microstructure can further be precisely tailored, and its interphase relationships controlled. In this regard, mechanics and material processing sciences are now playing a closer role together, both in the development and the application of composites.

As mentioned earlier, structural fiber composites is but one possible class of multiphase materials engineered with controlled microstructures. Other materials of current research interest include ceramics and metal matrix composites, chopped fiber and particulates composites, and concrete. It is most certain that other multiphase materials with heretofore unimaginable microstructures will be developed to provide even greater stiffness and strength; to control acoustical and vibrational damping; to improve resistance to impact loading and hostile environments, etc. As an example, the operating skin temperature on aircraft has increased drastically in recent years, Fig. 1. It is noted that there exists no material as of yet, which is suitable for the skin of the hypersonic transport of the future.

The phrase "mesomechanics" which is introduced here, is intended to describe an area of research that bridges the microstructure-property relationship studies of materials science with mechanics theories of a non-continuum character. This effort is in contrast with approaches that develop constitutive models of materials post-priori. Rather, the study of mesomechanics is aimed at developing the fundamental principles and the associated methodologies which can both guide the creation of multiphase materials with the desired microstructure and macroscopic properties, as well as support their efficient use in future structural systems.

The difficulties inherent in this new effort cannot be overstated. For what is proposed here will undoubtedly bring forth the need not only for modifications in the existing mechanics theories but also for establishment of new mechanics concepts, so as to outgrow traditional continuum concepts. This will create new research opportunities that require innovation in both analytical and experimental approaches. We believe, however, that the time has come for structural mechanics and materials science to embark together into a new era.

NEW THRUSTS IN MECHANICS-MATERIALS RESEARCH

Against this background, our planned research enhancement program will involve fundamental studies in the following general areas:

1. Constitutive Modeling of Multiphase Materials. In the past, continuum mechanics was remarkably successful in describing the effects of various loading and environmental factors on the deformation behavior of

structural materials. The fundamental postulate of continuum mechanics is that the influence domain of particle interactions is local, or at a range arbitrarily infinitesimal. It is will known that this postulate is at best approximate at microstructural levels from atomistic to granular scales. But the continuum concept has enabled mathematically rigorous development of modern mechanics theories which treat primarily what is called homogeneous materials. The application, however, becomes increasingly less acceptable with new multiphase materials, such as fiber composites, where the influence domain of phase interactions is nonlocal, or at a range several orders of magnitude larger than the size of the individual phases.

New constitutive modeling approaches are needed which not only describe the nonlocal effects but also the interactions associated with the material microstructure. It would seem essential that the microstructure be quantitatively characterized at the scale appropriate for determining the desired macro-properties; and that the characterization maintain sufficient geometric and mechanistic rigor, to enable the inclusion of the microstructural effects implicitly or explicitly in the mathematical development of material constitutive relations.

One can already see many difficulties that are inherent in this effort and will challenge our ingenuity. One basic problem is how to mathematically describe the complex shapes, orientations and distribution of the phases in three dimensions. Figure 2 depicts a sampling of a broad range of structural materials viewed on the dimensional scale of their property-determining microstructural features. It reveals the complexity of the challenge. Much pioneering work in mathematically describing complex three dimensional microstructures has been going on in the field of metallurgy. The evolution of microstructure in crystals, for instance, has been mathematically related to the minimization of anisotropic surface energies[5]; the geometric cell structure of porous media has been connected with the diffusion fluxes during the sintering process[6]. Application of fractal geometry to describe the apparent fracture surfaces of metallic[7] and brittle ceramics materials[8] opens up another possible avenue for developing generalized mathematical methods that can describe the three-dimensional character of microstructures.

Often, material microstructure changes with time. This happens, for example, during loading and unloading in which micro-damage in the form of cracks accumulates in the material and hence alters the characteristics of the microstructure. Thus, the microstructure-property characterization cannot be separated from the mechanics functions in the overall structural response determination. To relate such microstructural evolution effects through material constitutive relations to mechanics theories is the major challenge of mesomechanics. The cornerstone of this effort lies in the close collaboration of mechanics and materials researchers. As we proceed to formulate the constitutive relations for a multiphase material, we need to develop and control the process in which the desired microstructure can be generated. This will require collaboration between mechanics and materials science to a far greater extent than ever before.

2. Damage Mechanics of Multiphase materials. Continuum mechanics failure theories are mostly based on the stress state at a point; plasticity theories describe the formation of a yield zone near stress concentrations; dislocation theories identify weak slip lines or slip surfaces; and theories of fracture mechanics follow the crack tip and its propagation trajectory. All these failure theories treat local effects from a continuum point of view.

With multiphase materials, new concepts for material failure are also needed that can acount for the influencing factors at the phase scale. In addition to the microstructural details, one dominant factor is the efficiency of interface bonding between material phases. Experience with fiber composites has shown that the fiber/matrix interface bonding can be manipulated to change the initiation and coalescence mechanism of microcracks[9]. This in turn can yield a wide range of values for the apparent material toughness and other properties of the composite.

Microcracks at the phase scale often occur randomly in a body of large volume. This leads to statistically distributed failure and failure modes. But, development of distributed microcracks in multiphase materials, such as fiber composites, is also regulated by the microstructural geometry[10]. As microcracking is essentially a time-dependent process, during which loading or unloading is applied, the stability of the process determines the apparent failure of the material. To describe such processes and to establish criteria for stability would seem to require a combined statistical and mechanistic approach for what is being referred to as damage mechanics.

Clearly, to understand the initiation and growth mechanisms of damage at various dimensional scales

of the material microstructure presents another major challenge involving both mechanics and materials sciences.

3. Stress Waves in Multiphase Materials. Stress wave propagation has played an important role in modern technology. Applications have ranged from earthquake monitoring to material defect detection. Theories describing wave motions in solid bodies, however, have largely been developed on the basis of continuum mechanics principles, even though phenomena associated with material anisotropy and inhomogeneity have recently attracted increased attention.

With multiphase materials, new studies on such phenomena as nonlinear dispersion, energy dissipation, multiple scattering and surface wave effects will be needed, especially for stress waves of wave lengths comparable to the characteristic dimensions of the microstructure. Any such endeavor, however, would require an iterative approach in which the character of the sources, the mechanical properties of the material phases and the inherent microstructure are to be related; correlative studies involving analytical simulation and experimental measurement of both the standing and transient stress wave profiles are needed in order to properly delineate the true nature of the stress waves propagating in multiphase materials.

4 Very High Temperature Behavior. As mentioned earlier, there is an increasing demand for structural materials to function under very high temperature conditions and for prolonged duration. The wing skin of the hypersonic transport of the future, for example, must be able to retain its stiffness, strength, dimensional stability and other properties during long flights in a temperature environment of 3000^{0} F or more. Turbine engine components will operate at even higher temperatures. New material systems such as fiber reinforced metals and ceramics have the potential for satisfying both the high strength and high temperature requirements.

The mechanics applicable to these special materials is in an embryonic state of development. The principles of thermoelasticity for isotropic materials have been extended to composites and other anisotropic materials, and to inelastic state of deformation[11]. Recent unified constitutive theories of the phenomenological approach, such as the Bodner-Partom model, have incorporated temperature effects through internal variable provisions[12,13]. Microstructural mechanisms involved in elevated temperature effects are being examined in research on creep and fracture phenomena [14]. What is lacking is a basic understanding of the specific mechanisms of thermomechanical deformation and damage that occur at all levels of the material microstructure. An accounting for non-equilibrium thermodynamic processes under high strain rate and transient thermomechanical loading conditions is also needed.

Experimentation to identify the underlying mechanisms of thermomechanical response is perhaps the first important research need. This is a challenge of major proportion because elevated temperature is a hostile environment for laboratory equipment. Instruments and facilities for making in-situ measurements are lacking both in numbers and in precision. Nonetheless, efforts in this direction must accompany any development of advanced constitutive models that incorporate microstructural and damage effects.

The consequences of microstructural changes in response to high temperature loading to material property deterioration and damage accumulation need to be studied in order to provide a basis for reliable criteria of failure, such as evidenced in creep rupture and fatigue damage accumulation, and for material property tailoring during synthesis to achieve desired material performance requirements.

5. Nonlinear Structural Behavior. It is apparent that the mechanics of multiphase materials that account for microstructural effects will lead inherently to nonlinear behavior. The immediate consequence of nonlinearity is the loss of the applicability of the superposition principle. The response to a variety of loading or excitation conditions can no longer be synthesized from the individual effects of those conditions by simple addition. Another consequence of nonlinearity is the multiplicity of possible equilibrium configurations for a given set of operational conditions with varying degrees of stability attributes.

Both consequences have serious implications for all aspects of material application in structural system design. From a material characterization standpoint, nonlinear scale (size) effects must be considered in extrapolating test data or measurements from small size coupons to large scale components. These size effects need to be identified through basic research. In addition, time sequencing effects of different loading conditions must be understood if the complex dynamic loadings of real systems are to be amenable to analysis within mesomechanics.

The stability implication of nonlinear behavior of multiphase materials in structural system application remains an area of fundamental research. Recent studies on chaotic motions[15,16] in geometrically nonlinear systems have identified a variety of stability behaviors that have no counterpart in linear mechanics. With multiphase materials, nonlinearities of a constitutive origin, such as those manifested phenomenologically in hysteresis, strain hardening or softening, relaxation and creep, etc., must be examined from a system stability point of view.

6. Computational and Experimental Mechanics. The impact of mesomechanics studies on both the computational and the experimental mechanics will be extraordinary. The problems to be studied in mesomechanics will undoubtedly be nonlinear, time-dependent and three-dimensional in nature. The mathematical solutions to these kinds of problems pose an unprecedented challenge for computational mechanics. Even with the availability of high speed supercomputers, equipped with parallel-processor architectures, it may still be impractical if not impossible to perform the required analyses and simulations if a fundamental understanding of the physical problem is inadequate.

Although extensive research and development of modern computational methods has been conducted in nonlinear structural and solid mechanics in recent years[17], problems persist in accurately computing the true and complete nonlinear solutions. In finite element application, for instance, mesh refinement to improve numerical precision might compromise the inherent nonlinear effects accompanying the physical system. It seems that any future development of computational methods must involve insightful modeling, which truly represents the physics of the phenomenon to be simulated.

On the other hand, the physical quantities to be detected and measured, must be resolvable in a wide range of dimensional scales. Within the context of mesomechanics of multiphase materials, measurement in the micron and submicron scales may be necessary to resolve changes in the microstructures. Instruments and techniques are needed to measure the desired quantity nondestructively and in real-time, either by remote sensing or by in-situ monitoring. The high precision reqired, the very short response time and the hostile testing environment in which measurements are to be made all require novel developments in experimental mechanics.

CONCLUDING REMARKS

In this paper, we have outlined general research areas identified for enhanced attention in the structures and materials research program at AFOSR. Emphasis is placed on support of basic research in the interdisciplinary areas of mechanics and materials science. While the outlined research areas are more mechanistic in nature, as this paper is primarily directed toward the mechanics community, the ability of the mechanician and the material scientist to join efforts perform as a team is essential. As the task will require repeated and refined correlations between physical understanding and mathematical description of the same phenomena under consideration, the future work involved will be difficult to engage. But we believe that these outlined research areas represent an integral part of the emerging field of mesomechanics, which will be important to future progress in materials and structures technology. We hope that this discussion will serve as a focal point for a thorough and dedicated effort in this important research area.

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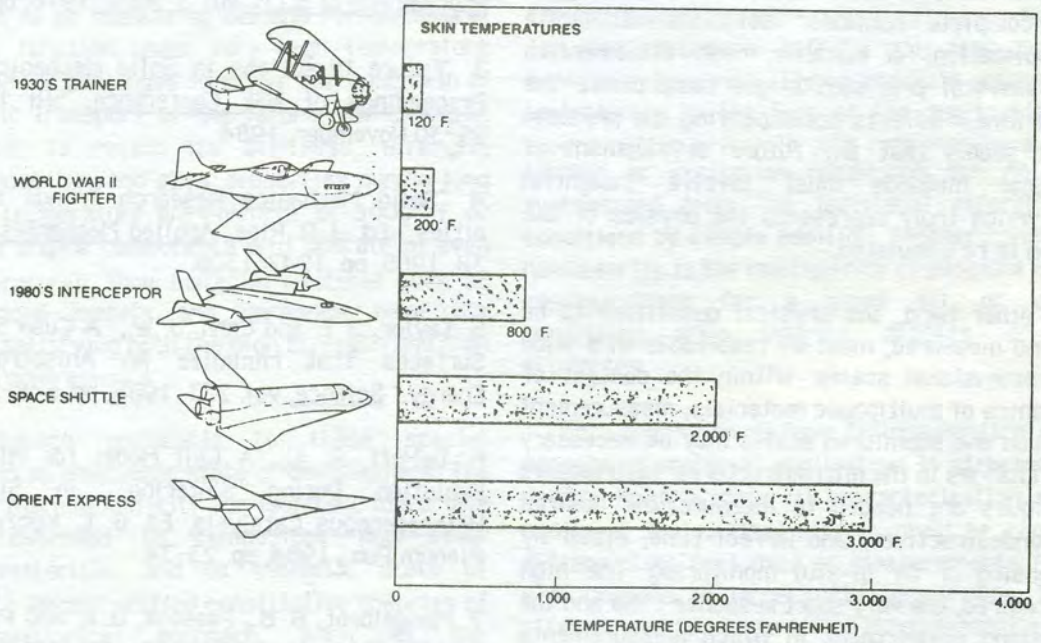
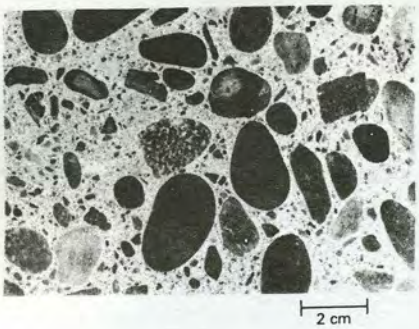
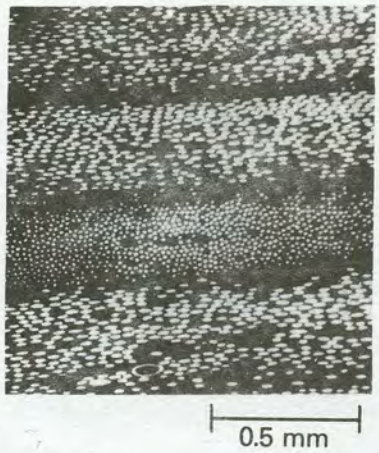


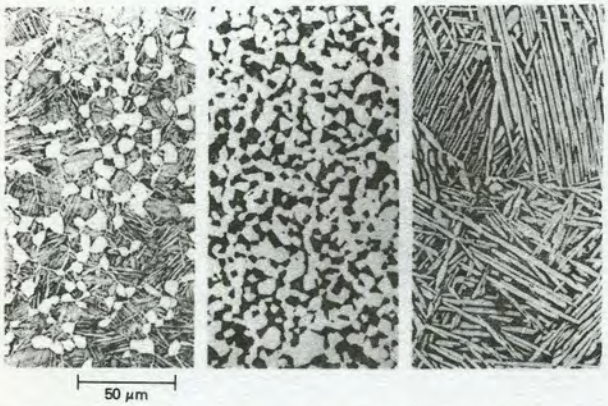
Fig. 1. Increasing Trend in Aircraft Skin Temperatures. Courtesy, Scientific American, vol. 255, 1986, p. 68.



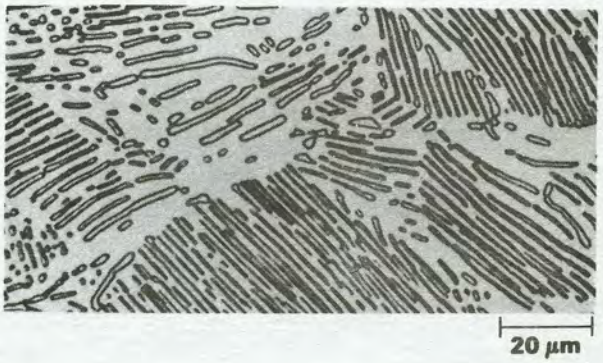
(a)Conventional Concrete: aggregate (dark) and cement paste (light). Courtesy, Concrete: Structures, Properties and Materials, Prentice-Hall, 1986, p. 19.



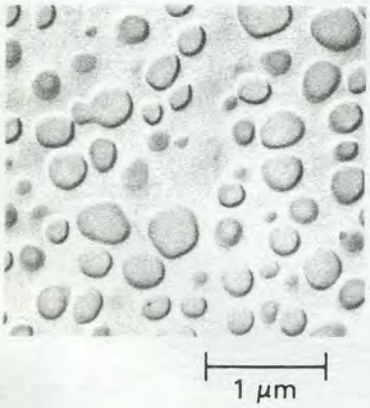
(b) Graphite-Epoxy Composite Laminate: AS graphite fibers (light) in 3501 epoxy matrix (dark). Courtesy, Metals Handbook, vol. 9, 1985, p. 593.



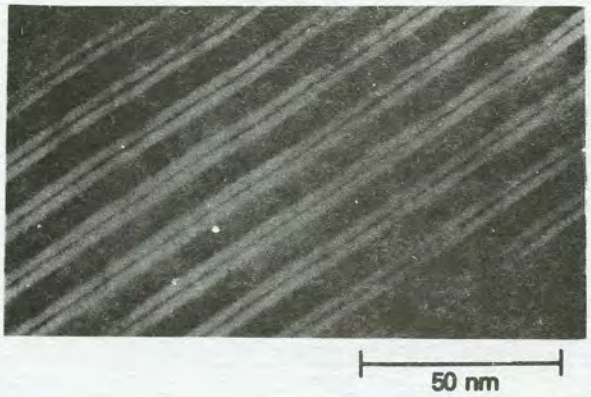
(c) Titanium Alloy Under Three Different Processing Conditions: alpha phase (light) in bata phase matrix (dark). Courtesy, Scientific American, vol. 255, 1986, p.163.



(d) Eutectoid Steel: iron carbite (dark) in ferrite matrix (light). Courtesy, Metals Handbook, vol. 9, 1985, p. 658.



(e) IN-738 Nickel-Based Superalloy: gamma prime particles in gamma matrix. Courtesy, Metals Handbook, vol. 9, 1985, p. 345.



(f) Artificially Structured Materials: three-layer super lattice built by molecular beam epitaxy. Courtesy, Scientific American, vol. 255, 1986, p. 134.

Fig. 2. A Sampling of Some Multiphase Materials Viewed at the Dimensional Scale of Their Phase Structural Features.